LIFING

Validation

Results comparison with Real Test Data

1 TABLE OF CONTENTS

2 INTRODUCTION

A mathematical validation of a computer code performing FEM based fatigue, i.e. Crack Initiation, analysis is, in general, not possible because of the following:

1. An unique fatigue analysis method does not exist.

   Many approaches have been developed over the years, and many will still be developed in the future:

   - multiple stress-based and strain-based fatigue parameters are available in literature;
   - critical plane or alternative approaches;
   - multiple cyclic plasticity models;
   - different damage cumulation models;
   - …

   A huge amount of papers are available in literature claiming the goodness of certain methods on the basis of adherence to dedicated experimental data. These highlight the fact some methods could be more appropriate than other depending on the problem being solved (type of geometry, loading modes, …) and/or material parameters (and in some cases also calibration factors) being used.

   (With *LIFING*, the analyst can choose among a list of many of the most acknowledged analysis techniques).

2. When a fatigue solver is used, a specific set of material parameters is adopted. However material fatigue data are characterized by a significant scatter. This means that, even if an ideal unique fatigue analysis parameter/approach would exist, results being produced would be 'deterministically' related to the specific material parameters used, therefore comparison with test data will inevitably show an 'error' due to the difference between the real material behaviour (of the used test specimen) and the modelled one.

3. The fatigue damage, or life, calculated on the basis of a FEM is strictly dependent on the FEM quality (mesh size, element type and order, boundary conditions, …). This means that two different FE Models of the same structure, solved with the same fatigue solver, using same approach and same material data, will show different fatigue results.

Because of the above, when a FEM assisted fatigue analysis is performed, the analyst must ensure that calculated results are valid within an acceptable error. This can be done if experimental data, for the problem being analysed, exist. The analyst can 'calibrate' results accordingly and apply appropriate scatter factors to the calculated results.

The *LIFING* 'validation' of the Crack Initiation capabilities provided in this document at section 1.1 must be looked from the perspective outlined above, that is:

- It is not a rigorous mathematical validation because a reference fatigue theoretical solution to be used for validating the implemented algorithms does not exist.
- Calculated results are based on analysis methods which are considered more appropriate, however any of the available methods can be used, which will deliver their own related results (which will be affected by the assumptions and limitations embedded in the method).
- Deviations between analysis and test data results must be judged considering appropriate scatter bands.

A different story is the validation of the Crack Growth capabilities.
This is because in literature multiple semi-empirical solutions exist, collected in different handbooks such as [7], [8], [9], and others.
The *LIFING* 'validation' of the Crack Growth capabilities are provided in section 1.2.
The validation shows how the Stress Intensity Factors calculated by *LIFING* for both 2D and 3D FEMs match reference solutions.

Additionally the integration of the growth model, da/dN, is compared to the AFGROW solution for an example of CG in a simple model with defined SIFs and a given variable amplitude stress sequence.

1 VALIDATION PROBLEMS

1.1 LIFING 'LIFE' MODULE

1.1.1 Tension-Torsion Loading of a Notched Shaft

1.1.1.1 Geometry and material

The Society of Automotive Engineers (SAE) Fatigue Design and Evaluation Committee coordinated an extensive testing program of a notched shaft subjected to tension and torsion loading. Results are documented in [2].

The used specimen geometry is shown in Figure 1-1.

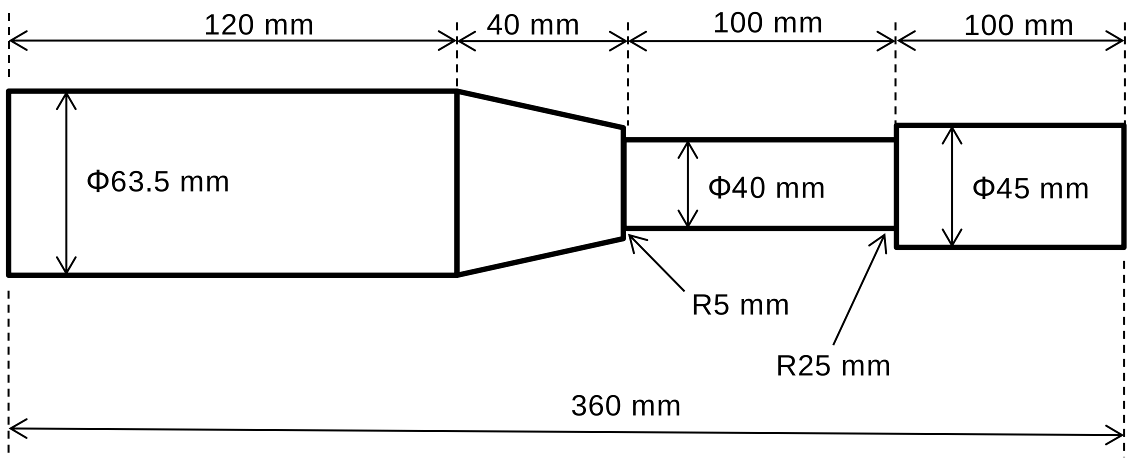


Figure 1-1 Specimen geometry

Material is steel SAE 1045 Hot rolled in normalized condition. Properties, from [1], are below reported:

Ultimate Stress *Su* = 612 MPa

Yield Stress = 380 MPa

E = 202000 MPa

Cyclic strength coefficient K' = 1258 MPa

Cyclic strain hardening exponent n' = 0.21

Fatigue strength coefficient σ_f' = 948 MPa

Fatigue ductility coefficient ε_f' = 0.26 MPa

Fatigue strength exponent b = -0.092

Fatigue ductility exponent c = -0.445

Fatigue shear strength coefficient τ_f' = 505 MPa

Fatigue shear ductility coefficient γ_f' = 0.413 MPa

Fatigue shear strength exponent b_γ = -0.097

Fatigue shear ductility exponent c_γ = -0.445

Figure 1-2 and Figure 1-3 show the relevant curves, and related equations, for the strain-based fatigue analysis ([1]): cyclic strain-stress curve and strain-life curve.

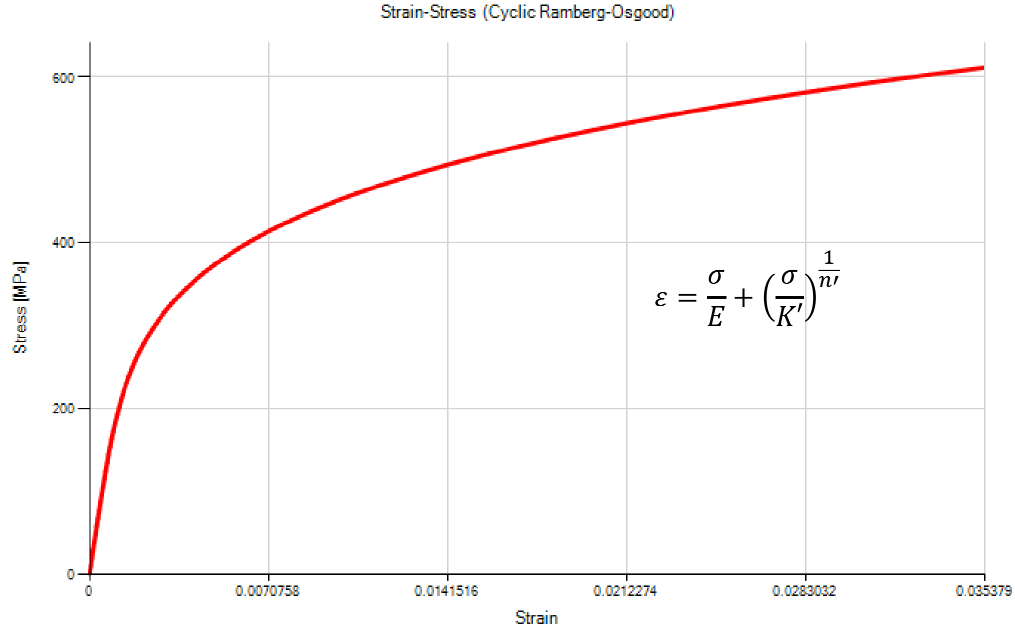


Figure 1-2 Cyclic Strain-Stress curve (cyclic Ramberg-Osgood equation)

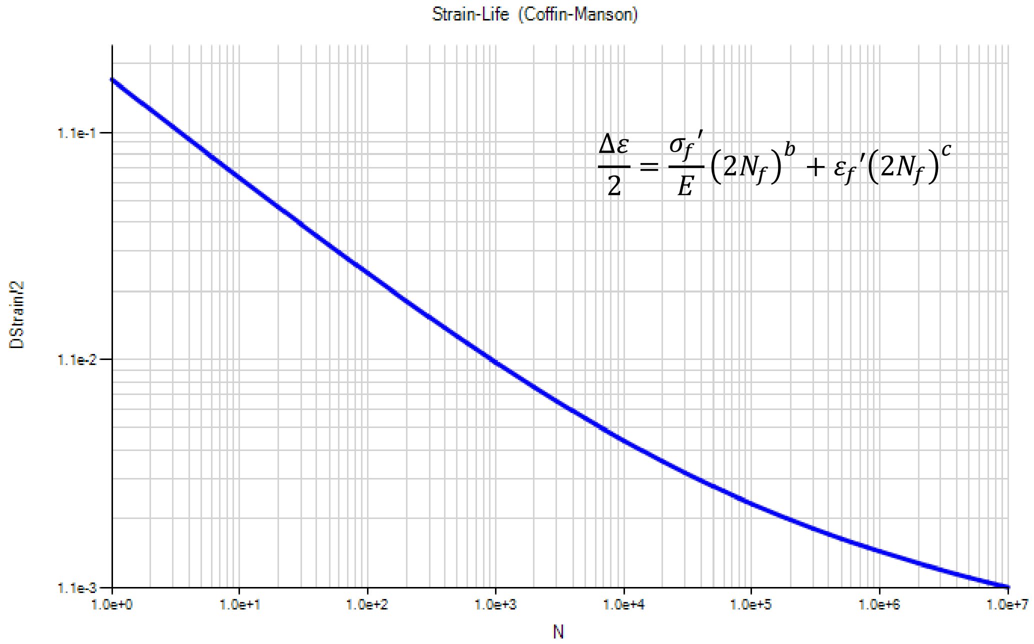


Figure 1-3 Strain-Life curve (Coffin-Manson equation)

[1] A strain-based fatigue analysis is considered because the specimen test campaign involves loads inducing local plasticity at the notch, i.e. at the 5mm radius.

As the specimen is in 'ground' surface conditions, the fatigue strength exponents are modified as follows:

$$b_{SF} = b - b_{red} = b - \frac{Log_{10}\left(\frac{1}{k_{SF}}\right)}{Log_{10}(2N_{SF})}$$

where, for ground surface conditions, the 'surface finish' factor is, from [3]

$$k_{SF} = 1.58 \cdot Su^{-0.085} = 0.916$$

As $N_{SF} = 10^6$, the factor b_{red} is 0.006.

The resulting fatigue strength exponents are:

b' = -0.092 – 0.006 = -0.098

b'_{γ} = -0.097 – 0.006 = -0.103

1.1.1.2 FEM description

Figure 1-4 shows the used FEM.

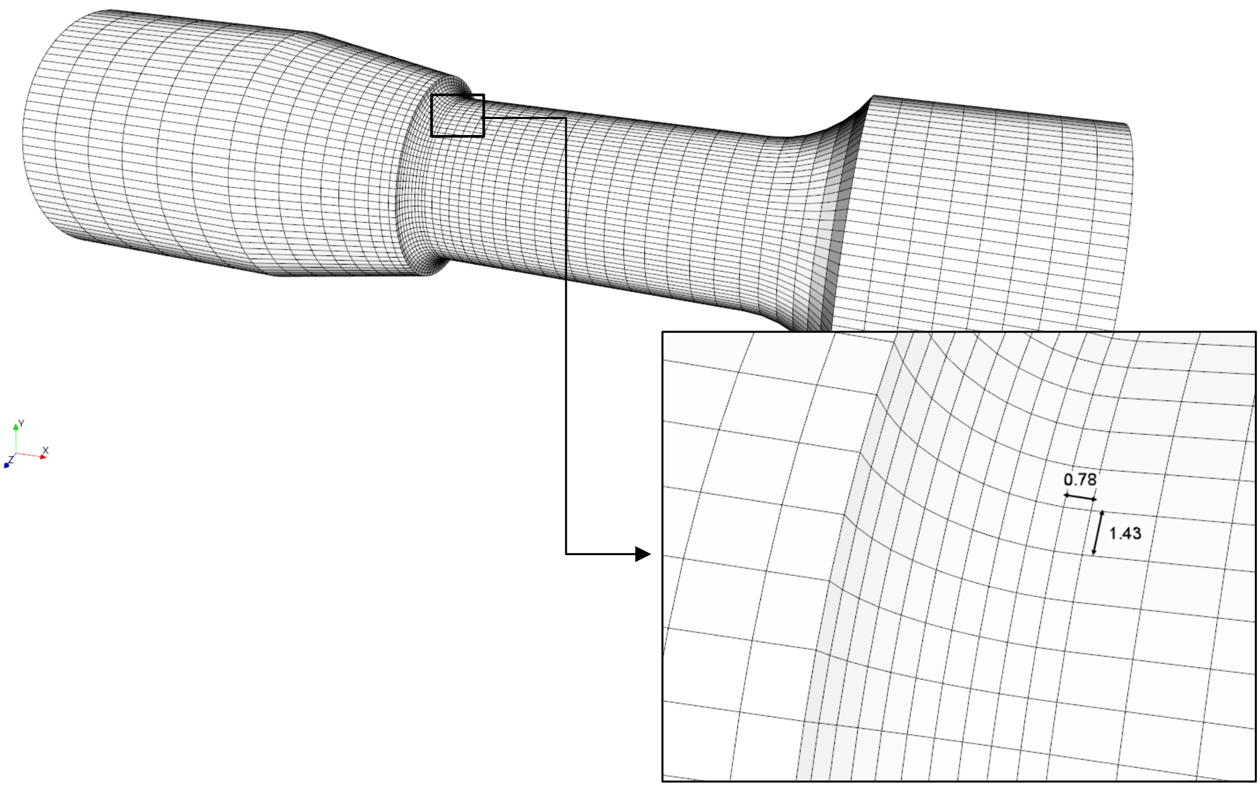


Figure 1-4 FEM

Used elements are 2nd order HEXA; at the notch the element size is 0.78x1.43 mm.
The model is ground constrained in the middle section of the left side cylindrical part and loaded at the middle section of the right side cylindrical part.

Two ‘unit’ loads are introduced (by means of rigid elements):

- LC 1: Force in Y direction of 6896.6 N at the right extremity, producing a Bending moment of 1000 Nm at the critical section (which is at 145 mm from the load application point)
- LC 2: Torsion 1000 Nm.

The solution, obtained using NASTRAN solver, for the two load cases are shown in Figure 1-5 and Figure 1-6.

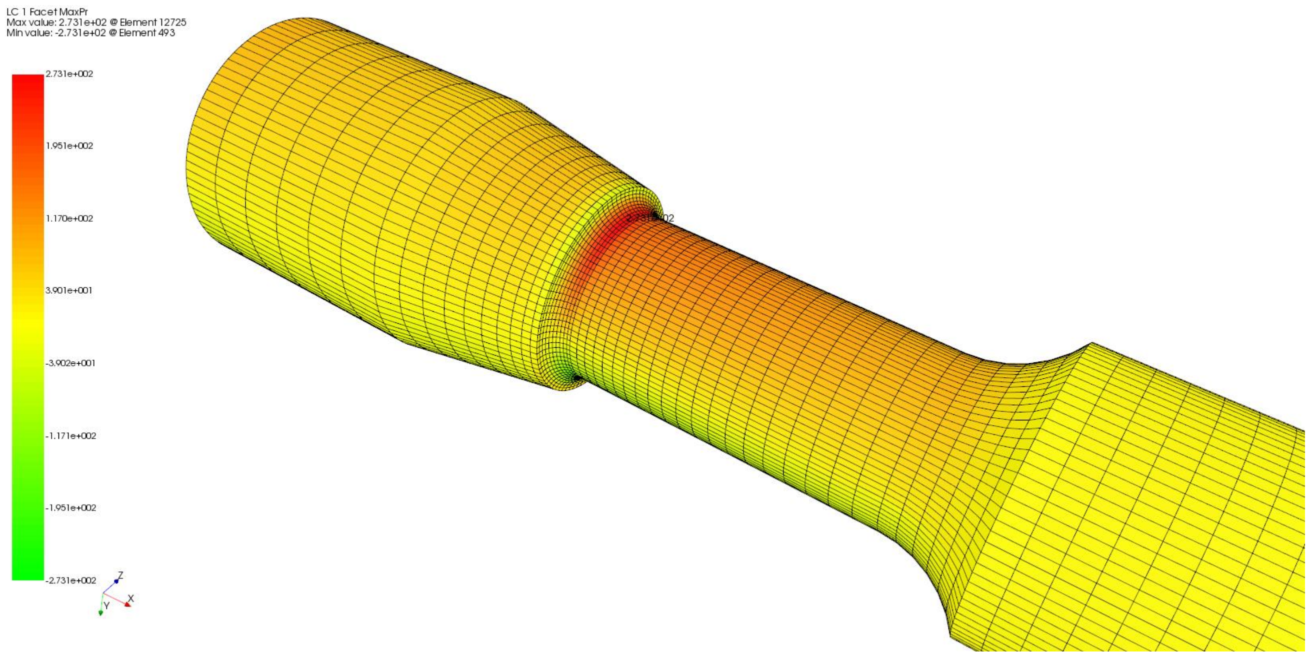


Figure 1-5 LC 1 solution – Max Principal Stress shown

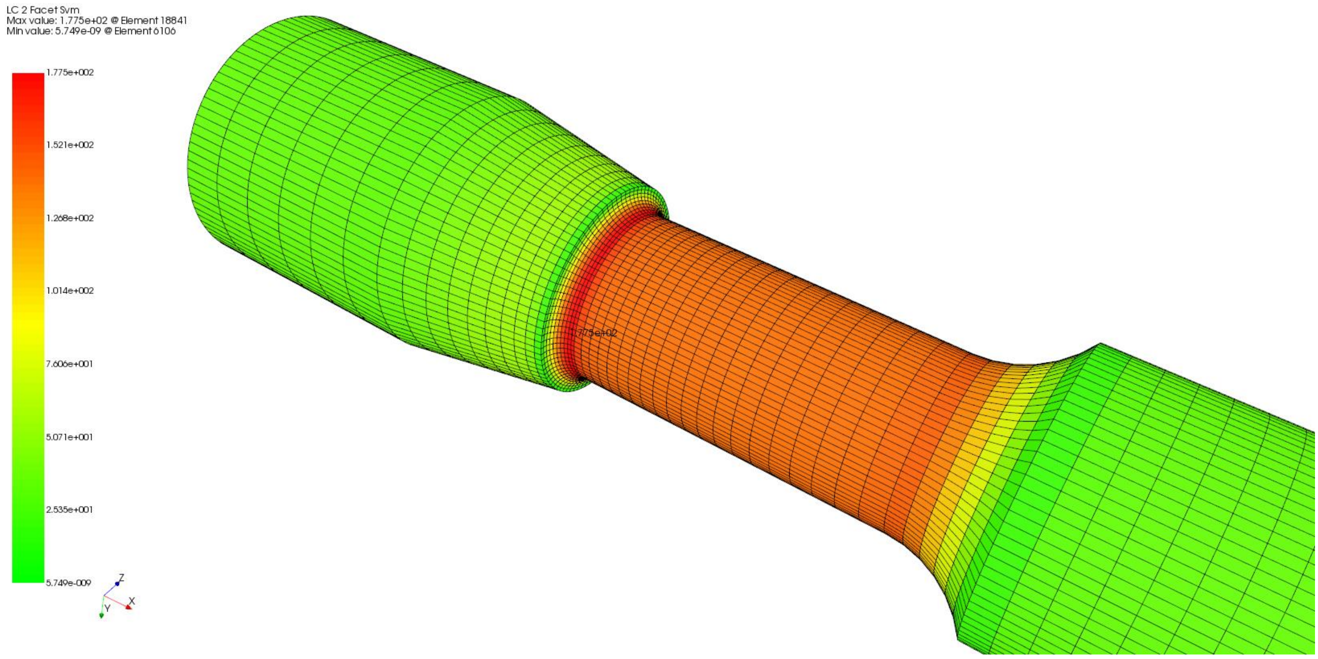


Figure 1-6 LC 2 solution – Von Mises Stress shown

The calculated nodal critical stresses at the notches are:

LC 1: Maximum Principal stress is 273.1 MPa (see Figure 1-7). To be noticed that the stress is multiaxial with positive biaxiality ratio $\sigma_{min}/\sigma_{max}$ = 55.5/273.1 = 0.203

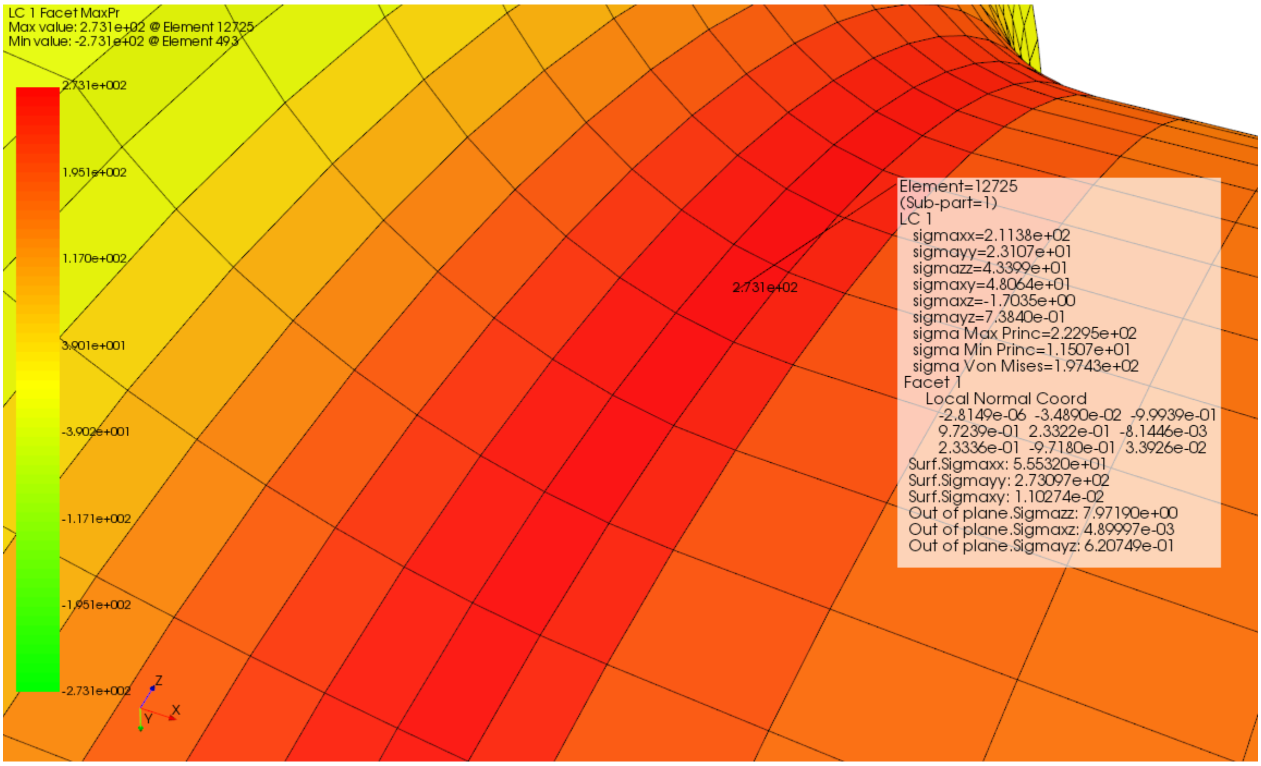


Figure 1-7 LC 1 solution – Element Stresses at critical location

LC 2: Maximum Von Mises stress is 177.5 MPa, i.e. max shear stress is 177.5/√3 = 102.5 MPa (see Figure 1-8). In this case the biaxiality ratio is $\sigma_{min}/\sigma_{max}$ = -1 (pure shear condition).

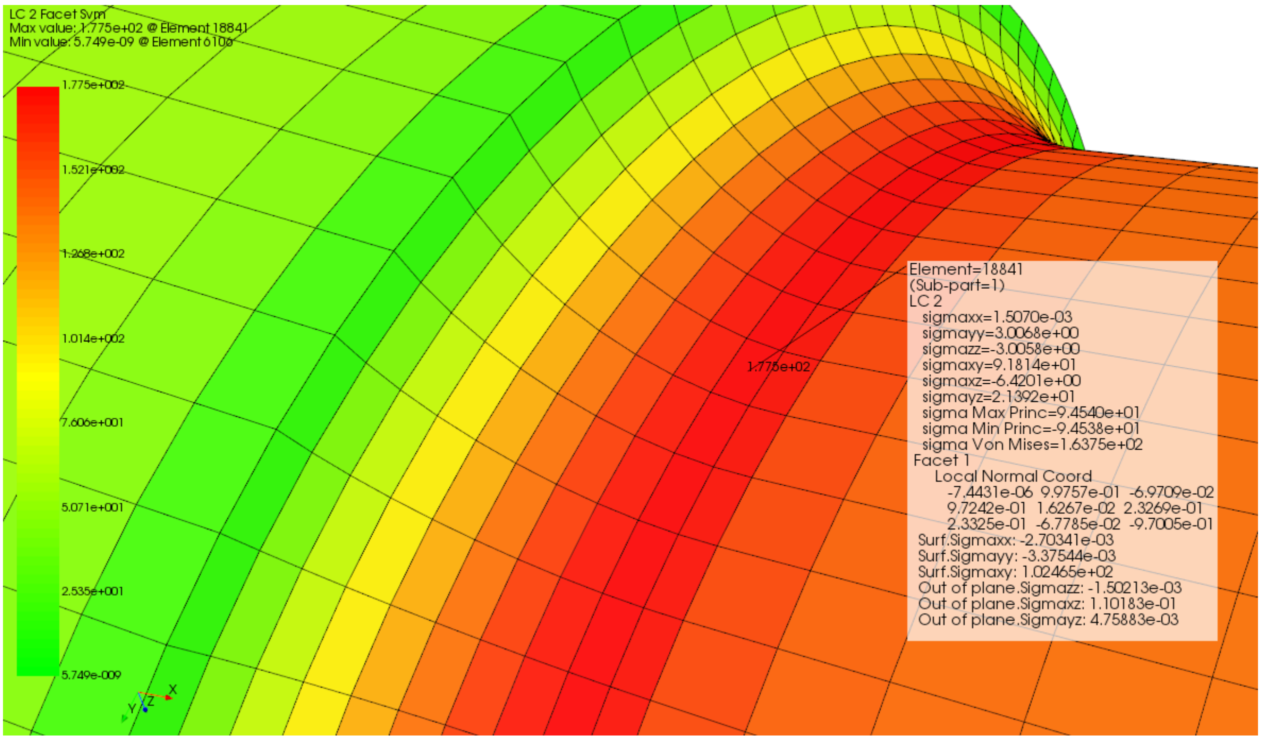


Figure 1-8 LC 2 solution – Element Stresses at critical location

1.1.1.3 Test and Analysis results

Constant loading tests are done (only bending, only torsion and combined bending and torsion). Results are shown below.

Identification	Bending Moment Nm	Torsion Moment Nm	Life to 1.0 mm crack cycles
JD-BR3-1	2800	0	2571
IL-BR3-2	2600	0	3000
AOS-BR3-1	2600	0	7930
JD-BR3-2	2600	0	8111
AOS-BR3-2	2586	0	14000
JD-BR2-1	1875	0	41360
BC-BR2-1	1875	0	55000
RN-BR2-1	1730	0	30000
IL-BR2-2	1730	0	49200
IL-BR2-1	1730	0	60000
AOS-BR2-1	1730	0	130000
AOS-BR2-2	1708	0	163800
AOS-BR1-1	1475	0	230000
AOS-BR1-2	1460	0	430000
JD-BR1-1	1475	0	464000
IL-BR1-1	1400	0	4494000
JD-TR3-1	0	3000	4057
IL-TR3-1	0	3000	7000
BC-TR3-1	80	2534	15000
BC-TR2-1	0	2400	65000
IL-TR2-1	0	2400	75700
GKN-TR1-1	0	2000	700000
RN-TR1-1	0	2000	750000
IL-TR1-1	0	2000	1584000
JD-TR0-1	0	1700	2324000
JD-TR0-2	0	1500	1515000
IL-XR3-1	1850	2550	2200
RN-XR3-1	1850	2100	4780
IL-XR3-3	1850	2100	6700
IL-XR3-1	1355	2550	5500
JD-XR3-1	2000	2100	5998
RN-XR2-1	1220	1700	60800
IL-XR2-1	1220	1710	72000
JD-XR2-1	1220	1710	107500
RN-XR1-1	990	1390	350000
IL-XR1-1	990	1390	933000
IL-XR1-1	725	1390	2000000
IL-YR2-1	1550	1090	80000
IL-YR2-2	1550	1090	97500
IL-YR3-1	2325	1350	2810
IL-YR3-2	2325	1350	3000
IL-YR3-1	1720	1350	17070
IL-YR3-2	1720	1350	21450
BC-YR2-1	1680	960	30000
JD-YR2-2	1680	900	84950
JD-YR2-1	1300	1400	84680
RN-YR1-1	1250	880	325000
IL-YR1-1	1250	880	600000
IL-YR1-1	1150	1090	229400

IL-YR1-2	1150	1090	238100
IL-YR1-1	920	880	3473000
IL-ZR3-1	1150	2700	3000
JD-ZR3-1	1250	2700	6402
IL-ZR3-1	851	2700	9000
IL-ZR3-2	840	2700	10000
IL-ZR2-1	780	2180	70000
IL-ZR2-2	780	2180	70680
IL-ZR2-3	570	2180	76100
IL-ZR2-4	570	2180	99560
JD-ZR2-1	845	1800	259900
IL-ZR1-1	460	1760	2350000
IL-ZR1-1	460	1760	3027000

It is to be noticed that a certain scatter, in some cases significant, is shown for tests with same loads.

LIFING is used to run the analysis for those tests that are done minimum twice.

Tests with only bending applied is solved by using the Smith-Watson-Topper fatigue parameter

$$\left[\sigma_N \cdot \frac{\Delta\varepsilon}{2}\right]_{max} = \frac{{\sigma'_f}^2}{E}\left(2N_f\right)^{2b'} + \sigma'_f \cdot \varepsilon'_f\left(2N_f\right)^{b'+c}$$

and cyclic plasticity is solved using Dowling method.

Tests with torsion or combined bending and torsion is solved with Fatemi-Socie fatigue parameter

$$\left[\frac{\Delta\gamma}{2}\left(1 + S\frac{\sigma_{N,max}}{S_y}\right)\right]_{max} = \frac{\tau'_f}{G}\left(2N_f\right)^{b'_\gamma} + \gamma'_f\left(2N_f\right)^{c_\gamma}$$

using fitting coefficient S = 1. Cyclic plasticity is solved in these cases with the Köttegen-Barkey-Socie Pseudo-Material method.

The table below and Figure 1-9 shows the comparison experimental VS analytical results.

Identification	Bending Moment N-m	Torsion Moment N-m	Life to 1.0 mm crack cycles	Lifing result
IL-BR3-2	**2600**	0	3000	5204
AOS-BR3-1	**2600**	0	7930	5204
JD-BR3-2	**2600**	0	8111	5204
JD-BR2-1	**1875**	0	41360	25224
BC-BR2-1	**1875**	0	55000	25224
RN-BR2-1	**1730**	0	30000	38299
IL-BR2-2	**1730**	0	49200	38299
IL-BR2-1	**1730**	0	60000	38299
AOS-BR1-1	**1475**	0	230000	91514
JD-BR1-1	**1475**	0	464000	91514
JD-TR3-1	0	**3000**	4057	13364
IL-TR3-1	0	**3000**	7000	13364
BC-TR2-1	0	**2400**	65000	92505
IL-TR2-1	0	**2400**	75700	92505

GKN-TR1-1	0	**2000**	700000	382716
RN-TR1-1	0	**2000**	750000	382716
RN-XR3-1	**1850**	**2100**	4780	4639
IL-XR3-3	**1850**	**2100**	6700	4639
RN-XR2-1	**1220**	**1700**	60800	40766
IL-XR2-1	**1220**	**1710**	72000	40766
JD-XR2-1	**1220**	**1710**	107500	40766
RN-XR1-1	**990**	**1390**	350000	215597
IL-XR1-1	**990**	**1390**	933000	215597
IL-YR2-1	**1550**	**1090**	80000	33618
IL-YR2-2	**1550**	**1090**	97500	33618
IL-YR3-1	**2325**	**1350**	2810	2694
IL-YR3-2	**2325**	**1350**	3000	2694
IL-YR3-1	**1720**	**1350**	17070	13973
IL-YR3-2	**1720**	**1350**	21450	13973
RN-YR1-1	**1250**	**880**	325000	187463
IL-YR1-1	**1250**	**880**	600000	187463
IL-YR1-1	1150	1090	229400	224452
IL-YR1-2	1150	1090	238100	224452
IL-ZR2-1	**780**	**2180**	70000	42948
IL-ZR2-2	**780**	**2180**	70680	42948
IL-ZR2-3	**570**	**2180**	76100	67677
IL-ZR2-4	**570**	**2180**	99560	67677

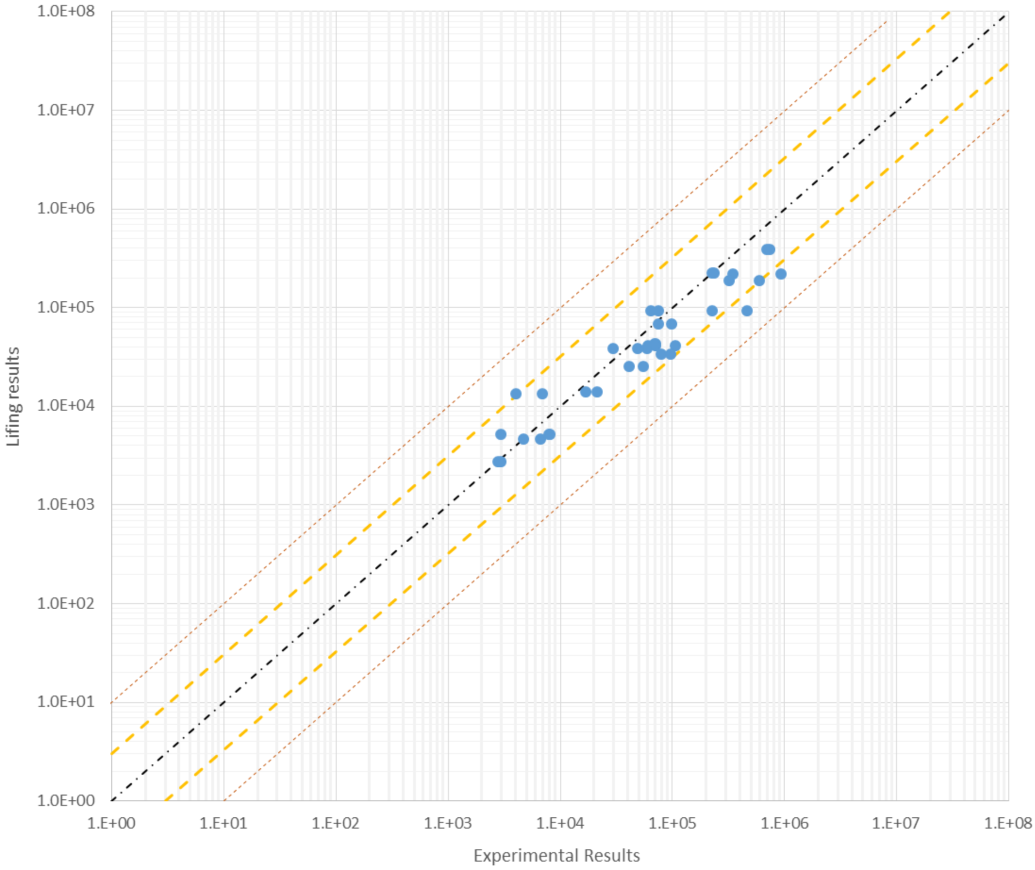


Figure 1-9 Experimental VS Analytical results

It is seen that *LIFING* results are in general, apart for few cases, within a scatter band of 3, which is a conventional band for fatigue good results.

1.2 LIFING 'GROWTH' MODULE

The Crack Growth results calculated by LIFING are validated in two steps:

1. The calculated Stress Intensity Factors are compared to the reference solutions, if available (e.g. Newman-Raju solutions).
2. The CG Life curve, resulting from the integration of da/dN model, for a given geometry which is reproducible in AFGROW is compared to the solution delivered by AFGROW.

1.2.1 Validation of SIFs

1.2.1.1 SIFs in 2D models

Case 1: SIFs in a finite plate subjected to constant stress

Figure 1-10 shows the analyzed case, characterized by the following data.

Width = 100 mm

Height = 200 mm

Thickness = 1 mm

Applied constant stress = 100 MPa

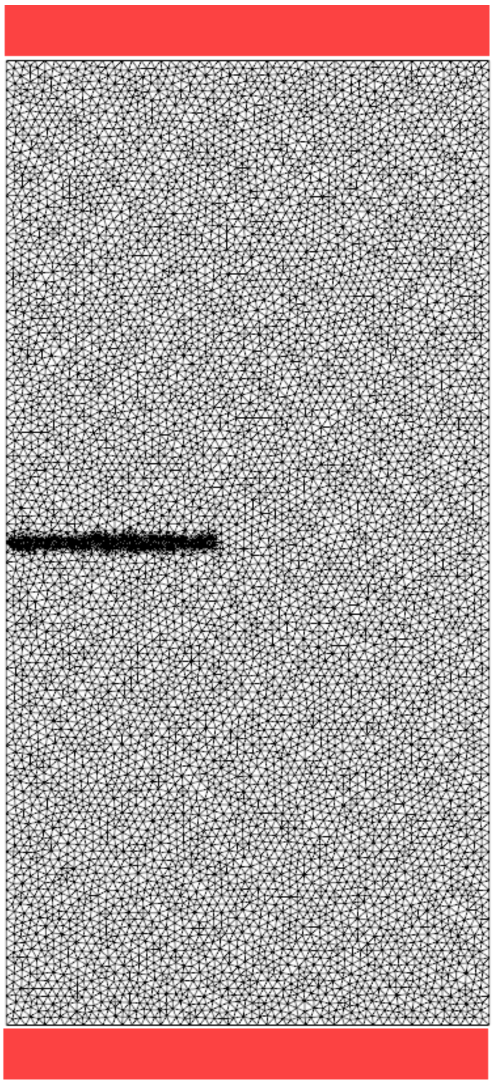


Figure 1-10 Test case 1

The model is generated by *LIFING* with the Quick2DFEM module.

The crack is propagated with the integrated module (which includes a 2D FEM mesher), where at the crack tip, quarter point elements are provided (yellow in Figure 1-11). .

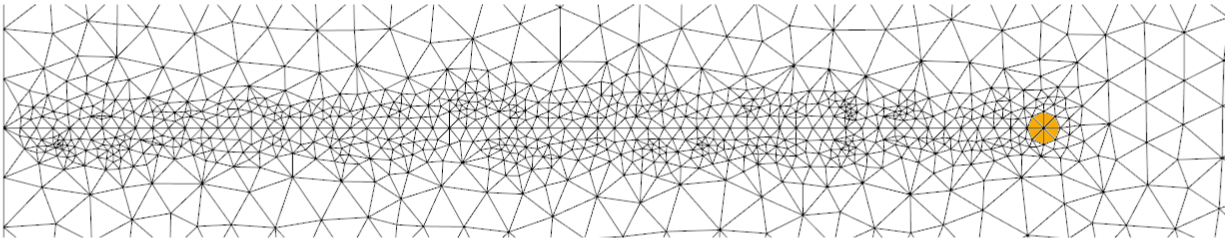


Figure 1-11 Zoom of the mesh at the crack propagation region

The *LIFING* solution is compared to the semi-empirical solution given in [7]. Maximum differences are, with the adopted mesh, in the order of 1.5%.

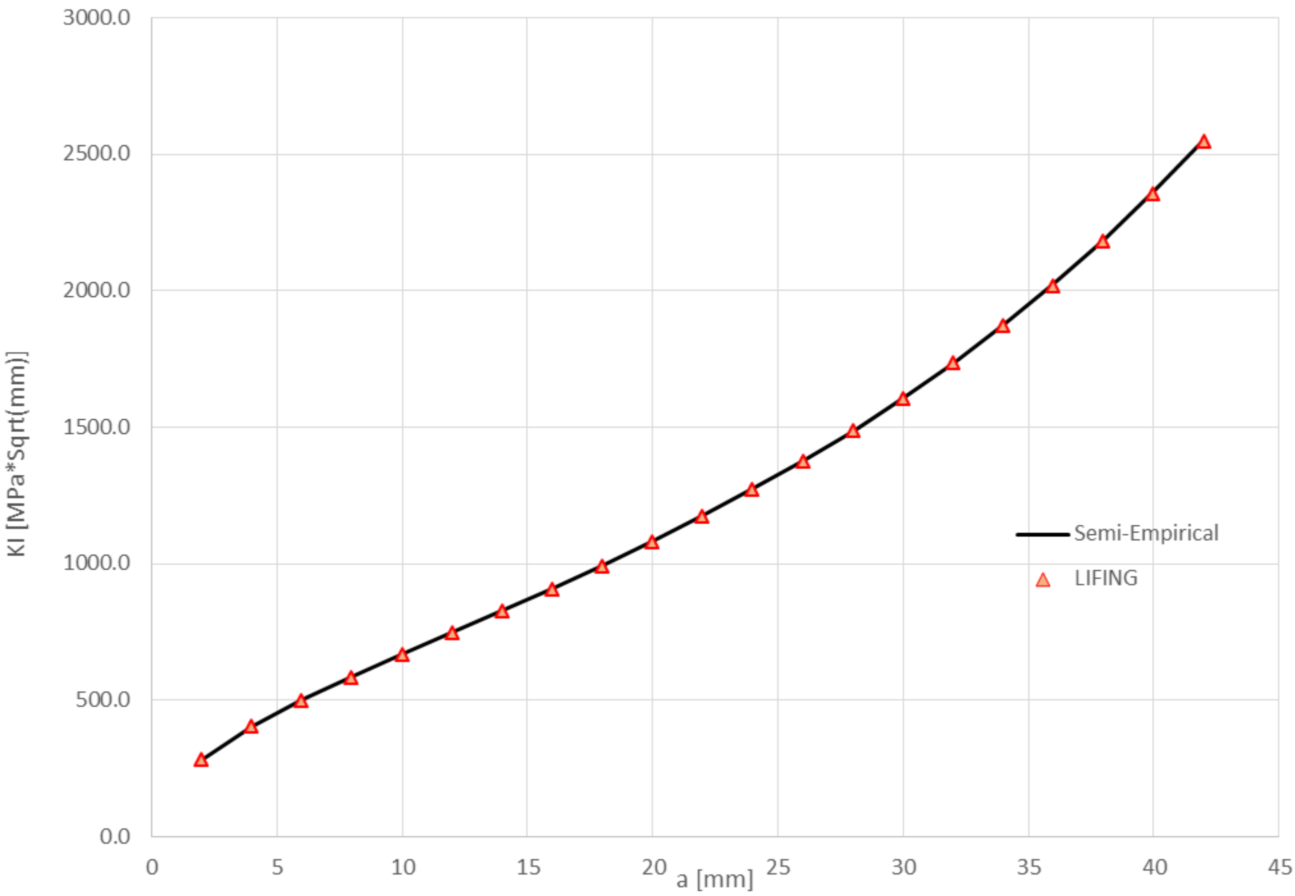


Figure 1-12 Test case 1 - comparison

Case 2: SIFs in a finite plate subjected to bending-in-plane stress

Figure 1-12 shows the analyzed case, characterized by the following data.

Width = 100 mm

Height = 200 mm

Thickness = 1 mm

Applied stress = from 100 MPa to -100 MPa

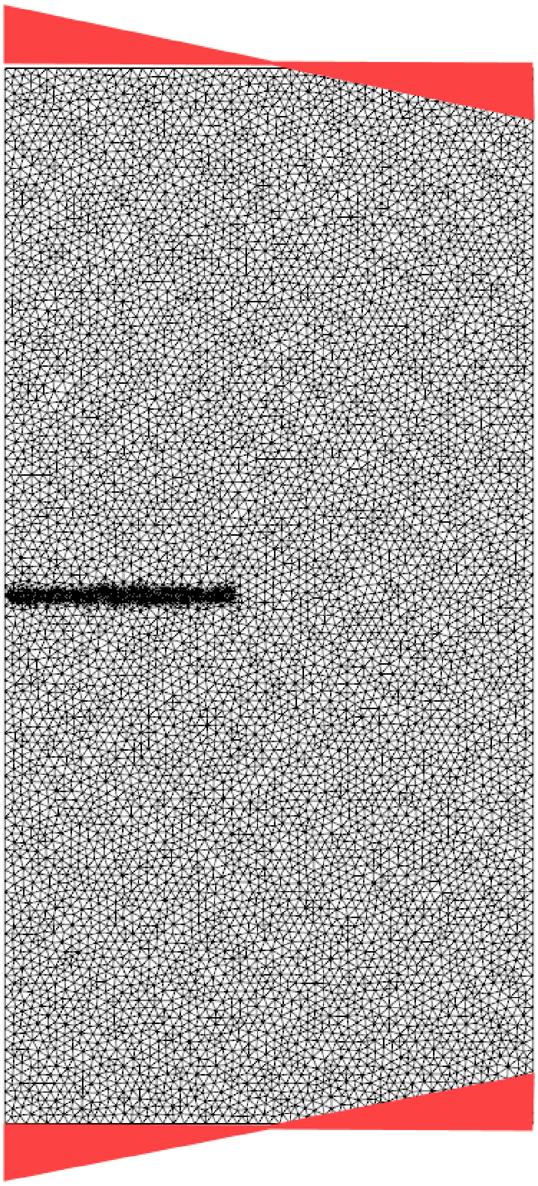


Figure 1-13 Test case 2

The *LIFING* solution is compared to the semi-empirical solution given in [7]. Maximum differences are, with the adopted mesh, in the order of 2%.

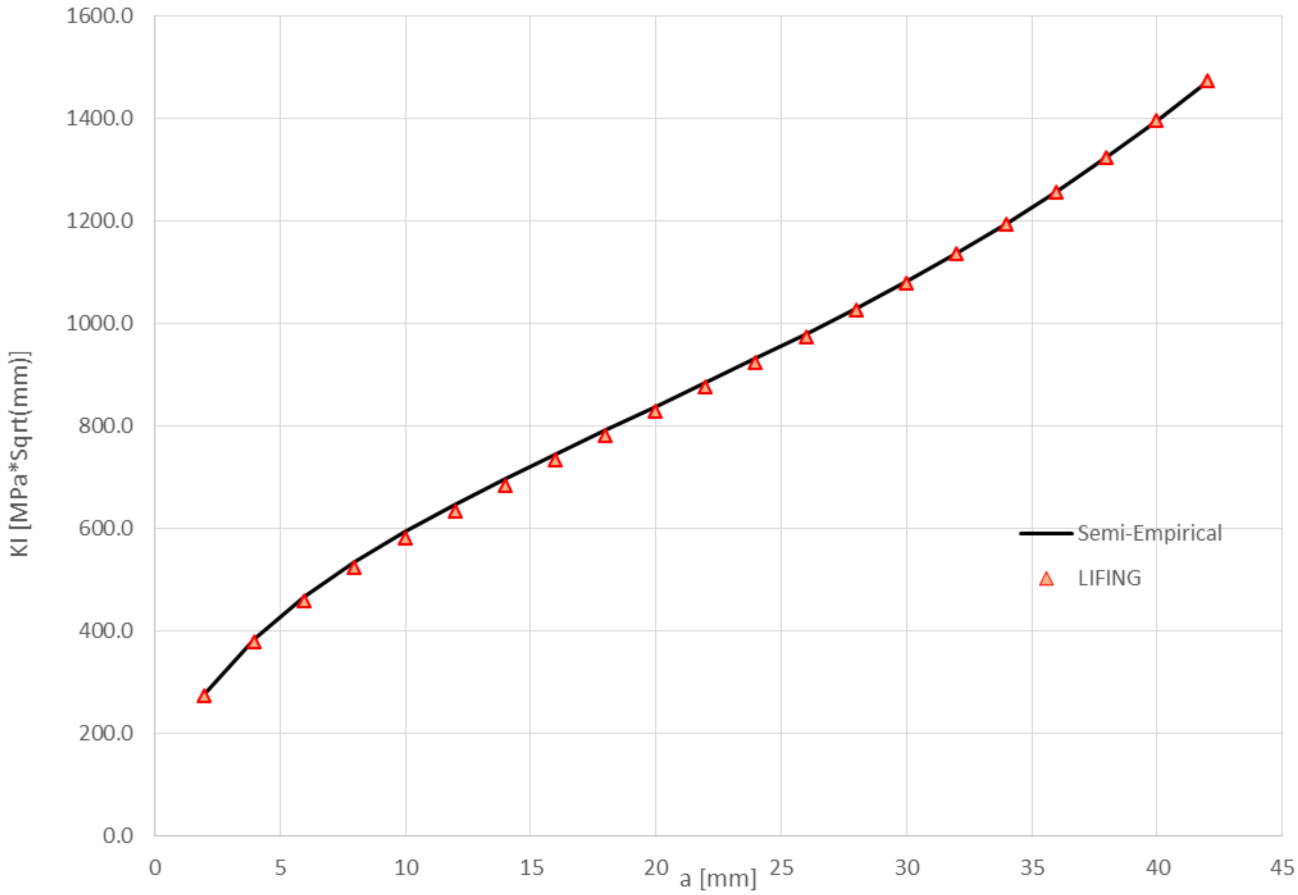


Figure 1-14 Test case 1 - comparison

1.2.1.2 SIFs in 3D models

A 3D model is built with the module Quick2DFEM, shown in Figure 1-15.

Width = 50 mm

Depth = 200 mm

Thickness = 10 mm

Applied stress = 150 MPa

Two circular cracks are calculated:

Crack 1: a = c = 2 mm

Crack 2: a = c = 5 mm

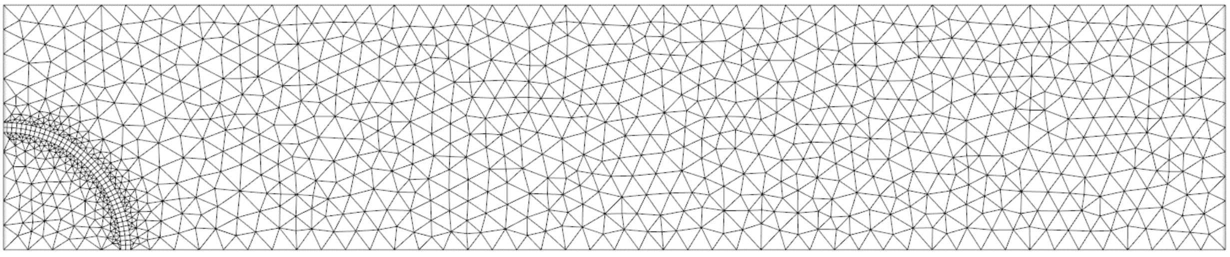


Figure 1-15 Test case 3

The *LIFING* solution, calculated with M-Integral, is compared to the corresponding Newman-Raju semi-empirical solution provided in [9].

It is seen in Figure 1-16 that the maximum deviation is 1.5%.

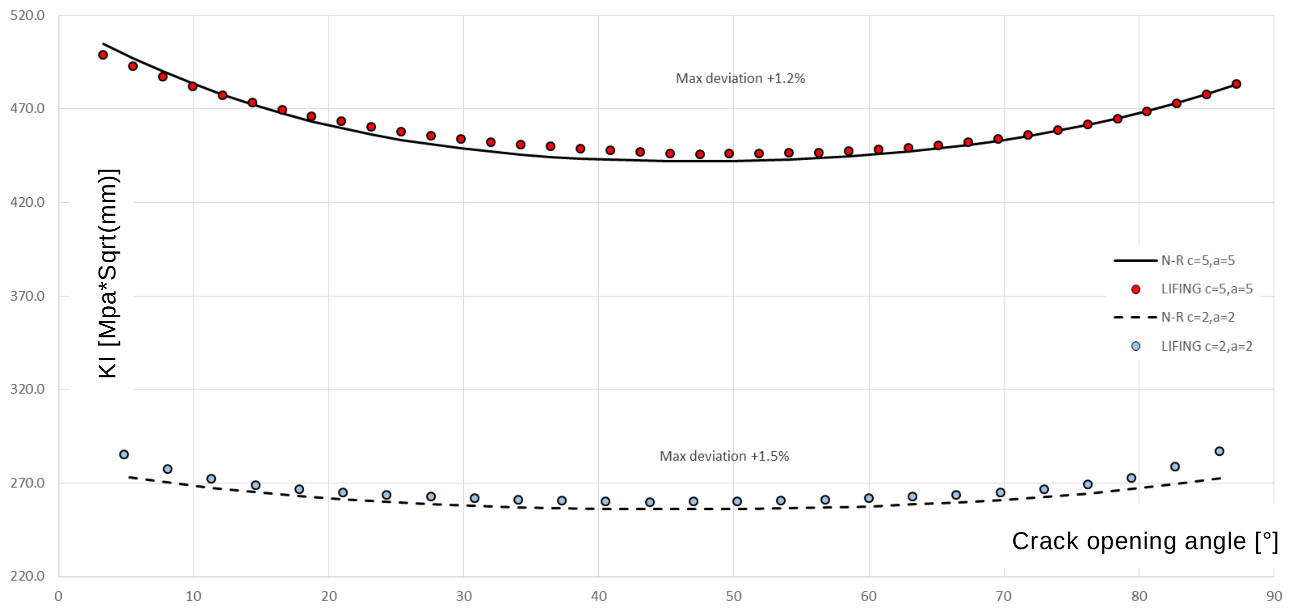


Figure 1-16 Test case 3 – comparison

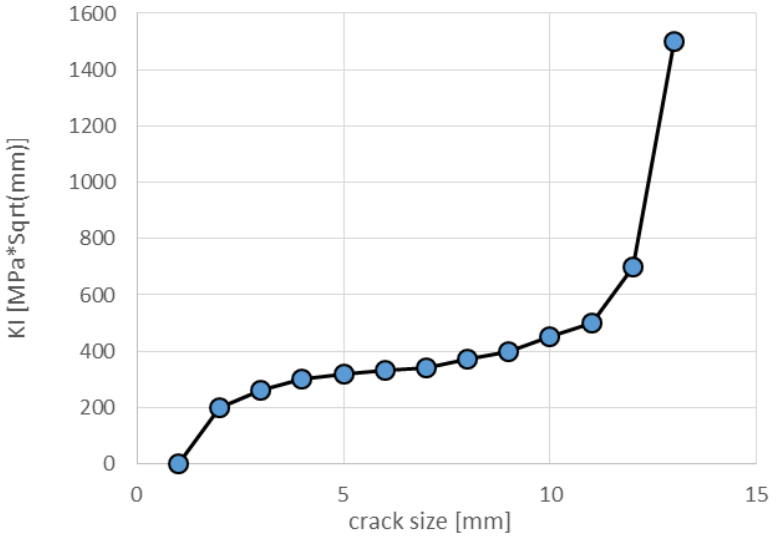
1.2.2 Validation of the da/dN integration

LIFING-Growth module can integrate da/dN given based on the NASGRO equation as well as tabular format.

Here the validation is given based on the first type.

The following SIFs are considered.

The adopted spectrum is the normalized FALSTAFF, i.e. made of 35966 events (variable amplitude), representative of stress levels encountered by a jet aircraft in 200 flights. The adopted material is 2124-T851 PLT & SHT; T-L; LA,HHA, from the NASGRO database.



c [mm]	SIF [MPa*Sqrt(mm)]
0.5	200
1	260
2	300
3	320
4	330
5	340
10	370
15	400
20	450
22	500
25	700
30	1500

Table 1-1 SIFs

The comparative result (*LIFING* vs AFGROW) is provided in Figure 1-17, which shows the perfect match ([2]).

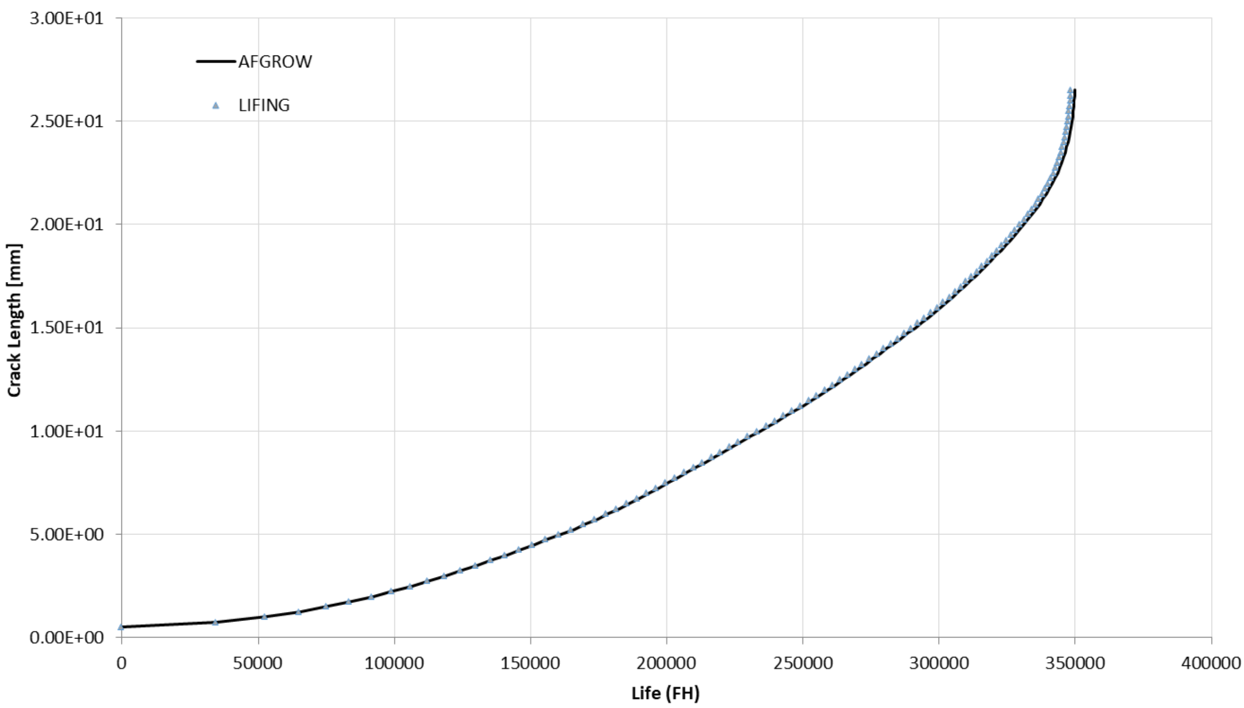


Figure 1-17 da/dN integration - comparison

[2] The LIFING run is performed with the option to integrate the SIFs, similarly to AFGROW, from the corresponding Compliance function (see *LIFING* Technical Reference).

2 REFERENCES

[1] Multiaxial Fatigue – D.F. Socie, G.B. Marquis

[2] Multiaxial Fatigue: Analysis and Experiments, AE-14, G.E. Leese, D.F. Socie, Society of Automotive Engineers, Warrendale, PA, 1989

[3] Allowable Working Stresses, Noil and Lipson, Society for Experimental Stress Analysis, Vol III, no. 2

[4] Stress Intensity Factor Equations for Cracks in Three-Dimensional Finite Bodies Subjected to Tension and Bending Loads, Newman and Raju

[5] Development of Weight Functions and Computer Integration Procedures for Calculating Stress Intensity Factors Around Cracks Subjected to Complex Stress Fields, Glinka

[6] Weight Functions and Stress Intensity Factors for Corner Quarter-Elliptical Crack in Finite Thickness Plate Subjected to In-Plane Loading

[7] The Stress Analysis of Cracks handbook, H.Tada, P.C. Paris, G.R. Irwin

[8] Evaluation of Stress Intensity Factors, D.P. Rook, D.J. Cartright

[9] Stress Intensity Factor Equations for Cracks in Three-Dimensional Finite Bodies Subjected to Tension and Bending Loads, Newman and Raju

[10] AFGROW Technical Reference